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Study of $B_s \to D_{sJ}(2317, 2460)l\bar{\nu}$ semileptonic decays in the CQM model

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Abstract. Assuming $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ to be the $(0^+, 1^+)$ chiral partners of the regular particles $D_s(1968)$ and $D^*_s(2112)$, we calculate the semileptonic decays of B_s to $D_s(1968)$, $D^*_s(2112)$, $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ in terms of the constituent quark meson model. The large branching ratios of the semileptonic decays of B_s to $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ indicate that these two semileptonic decays should be seen in future experiments.

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1 Introduction

The discoveries of the exotic mesons $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ [1–4], whose spin–parity structures are respectively 0^+ and 1^+, have attracted great interest of both theorists and experimentalists in high energy physics. $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ are supposed to be $(0^+, 1^+)$ chiral partners of D_s and D^*_s [5–7], i.e. p-wave excited states of D_s and D^*_s [8]. Beveren and Rupp suggested that $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ are made up of c and $\bar{s}$, a suggestion based on a study of the mass spectra [9, 10]. With QCD spectral sum rules, Narison calculated the masses of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ by assuming them to be quark–antiquark states and obtained results that are consistent with the experiment data within a wide error range [11]. Very recently, considering the contribution of the DK continuum in QCD sum rules, Dai et al. obtained the mass of $D^*_{sJ}(2317)$, which is consistent with the mass given by experiment [12]. Meanwhile, some authors suggested that $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ may have a four-quark structure [13–18]. Thus, one needs to use various theoretical approaches to clarify the mist surrounding the structures of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$. The studies of the production and decay of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ are very interesting topics.

The semileptonic decays of B_s are among the ideal platforms to study the production of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$. Especially the large hadron collider (LHC) will be run in 2007, which can produce large amounts of data on B_s. Thus, measurements of $B_s \to D_{sJ}(2317, 2460)l\bar{\nu}$ would be realistic. In [19], the author calculated $B_s \to D_{sJ}(2317, 2460)l\bar{\nu}$ decays by QCD sum rules in HQET. Recently, the authors of [20, 21] completed the calculations of $B_s \to D_{sJ}(2317, 2460)l\bar{\nu}$ semileptonic decays by QCD sum rules and obtained large branching ratios. However, the results obtained by [20, 21] are one order smaller than those given by [19]. Thus, studies on $B_s \to D_{sJ}(2317, 2460)l\bar{\nu}$ with other plausible models would be helpful. It not only will deepen our understanding of the properties of these states but also test the reliability of models that are applied to calculate the semileptonic decays.

In this work, we study the $B_s \to D_s(1968)l\bar{\nu}$, $B_s \to D^*_s(2112)l\bar{\nu}$ and $B_s \to D_{sJ}(2317, 2460)l\bar{\nu}$ semileptonic decays in terms of the constituent quark meson (CQM) model. In order to complete the calculations of the semileptonic decays of B_s, we firstly base ourselves on the assumption that $(D_s(1968), D^*_s(2112))$ with spin–parity $(0^-, 1^-)$ and $(D^*_{sJ}(2317), D_{sJ}(2460))$ with spin–parity $(0^+, 1^+)$ can be respectively categorized as $H(0^-, 1^-)$ and $S(0^+, 1^+)$ doublets in HQET.

The CQM model was proposed by Polosa et al. [22] and has been well developed later in the work of Ebert et al. [23–25] (see [22] for a review). The model is based on an effective Lagrangian that incorporates flavor–spin symmetry for heavy quarks with chiral symmetry for the light quarks. Employing the CQM model to study the phenomenology of heavy meson physics, reasonable results have been achieved [26–32]. Therefore, we believe that the model is applicable to our processes and expect to get relatively reliable conclusions.

This paper is organized as follows. After the introduction, in Sect. 2, we formulate the semileptonic decays of B_s to $D_s(1968)$, $D^*_s(2112)$, $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$. The numerical results along with all the input parameters are presented in Sect. 3. Section 4 is devoted to a discussion and the conclusion. Some detailed expressions are collected in the appendix.

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2 Formulation

For the convenience of the reader, we give a brief introduction to the CQM model [22]. The model is relativistic and is based on an effective Lagrangian that combines the HQET and chiral symmetry for the light quarks:

$$\begin{aligned}\mathcal{L}_{\rm CQM} = & \bar{\chi}\left[\gamma\cdot(\mathrm{i}\partial+V)\right]\chi+\bar{\chi}\gamma\cdot A\gamma_5\chi-m_q\bar{\chi}\chi \\ & +\frac{f_\pi^2}{8}\mathrm{Tr}\left[\partial^\mu\Sigma\partial_\mu\Sigma^+\right]+\bar{h}_v\left(\mathrm{i}v\cdot\partial\right)h_v \\ & -\left[\bar{\chi}\left(\bar{H}+\bar{S}+\mathrm{i}\bar{T}^\mu\frac{\partial_\mu}{\Lambda}\right)h_v+\mathrm{h.c.}\right] \\ & +\frac{1}{2G_3}\mathrm{Tr}\left[\left(\bar{H}+\bar{S}\right)(H-S)\right]+\frac{1}{2G_4}\mathrm{Tr}\left[\bar{T}^\mu T_\mu\right]\,,\end{aligned}$$

where the fifth term is the kinetic term of the heavy quarks with $\not{v}h_v = h_v$; H and S denote the super-fields corresponding to the doublets $(0^-, 1^-)$ and $(0^+, 1^+)$, respectively. The explicit matrix representations of H and S read [33]

$$H = \frac{1+\not{v}}{2}\left[P^*_\mu\gamma^\mu - P\gamma_5\right], \tag{1}$$

$$S = \frac{1+\not{v}}{2}\left[P^{*\prime}_{1\mu}\gamma^\mu\gamma_5 - P_0\right], \tag{2}$$

where P, $P^{*\mu}$, P_0 and P_1 are the annihilation operators of the pseudoscalar, vector, scalar and axial vector mesons, which are normalized as

$$\langle 0|P|M(0^-)\rangle = \sqrt{M_H}\,,\ \ \langle 0|P^{*\mu}|M(1^-)\rangle = \sqrt{M_H}\epsilon^\mu\,,$$
$$\langle 0|P_0|M(0^+)\rangle = \sqrt{M_S}\gamma_5\,,\ \langle 0|P_1^{*\mu}|M(1^+)\rangle = \sqrt{M_S}\gamma_5\epsilon^\mu\,.$$

T is the super-field corresponding to the doublet $(1^+, 2^+)$

$$T^\mu = \frac{1+\not{v}}{2\sqrt{2}}\left[P_2^{*\mu\nu}\gamma_\nu - \sqrt{\frac{3}{2}}P^*_{1\nu}\gamma_5\left(g^{\mu\nu}-\frac{1}{3}\gamma^\nu(\gamma^\mu - v^\mu)\right)\right]. \tag{3}$$

$\chi = \xi q (q = u, d, s)$ is the light quark field and $\xi = \mathrm{e}^{\frac{\mathrm{i}M}{f_\pi}}$; M is the octet pseudoscalar matrix. We also have

$$V^\mu = \frac{1}{2}\left(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger\right), \tag{4}$$

$$A^\mu = \frac{-g\mathrm{i}}{2}\left(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger\right). \tag{5}$$

Because the spin–parity of $D_s(1968)$ and $D_s^*(2112)$ are respectively 0^- and 1^-, $D_s(1968)$ and $D_s^*(2112)$ can be embedded into the H-type doublet $(0^-, 1^-)$, whereas $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ belong to the S-type doublet $(0^+, 1^-)$. Thus, we can calculate the semileptonic decays of B_s to $D_s(1968)$, $D_s^*(2112)$, $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$.

2.1 Calculations of $B_s \to D_s(1968)l\bar{\nu}$ and $B_s \to D_s^*(2112)l\bar{\nu}$ in the CQM model

The four fermion operator of $b \to c + l\bar{\nu}$ that is relevant to the semileptonic decays of the B_s to D_s mesons reads [34]

$$\mathcal{O} = \frac{G_{\rm F}V_{cb}}{\sqrt{2}}\bar{c}\gamma^\mu(1-\gamma_5)b\bar{\nu}\gamma_\mu(1-\gamma_5)l\,. \tag{6}$$

The transition amplitudes of $B_s \to D_s(1968)l\bar{\nu}$ and $B_s \to D_s^*(2112)l\bar{\nu}$ can be written as

$$\begin{aligned}\mathcal{M} &= \langle D_s^{(*)}l\bar{\nu}|\mathcal{O}|B_s\rangle \\ &= \frac{G_{\rm F}V_{cb}}{\sqrt{2}}\left\langle D_s^{(*)}|\bar{c}\gamma^\mu(1-\gamma_5)b|B_s\right\rangle\langle l\bar{\nu}|\bar{\nu}\gamma_\mu(1-\gamma_5)l|0\rangle\,,\end{aligned} \tag{7}$$

where the hadronic matrix element is related to nonperturbative QCD effects. In the HQET symmetries, the hadronic matrix element can be expressed in the following form:

$$\begin{aligned}&\langle D_s(v')|\bar{c}\gamma_\mu(1-\gamma_5)b|B_s(v)\rangle \\ &= \sqrt{M_{B_s}M_{D_s}}\left(v_\mu + v'_\mu\right)\xi(\omega)\,,\end{aligned} \tag{8}$$

$$\begin{aligned}&\langle D_s^*(v',\epsilon)|\bar{c}\gamma_\mu(1-\gamma_5)b|B_s(v)\rangle \\ &= \sqrt{M_{B_s}M_{D_s^*}}\left[\mathrm{i}\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}v'^\alpha v^\beta - (1+\omega)\epsilon^*_\mu \right.\\ &\quad\left. + (\epsilon^*_\mu\cdot v)v'_\mu\right]\xi(\omega)\,,\end{aligned} \tag{9}$$

where $\omega = v\cdot v'$. In HQET, $\xi(\omega)$ is the Isgur–Wise function, which is a dimensionless probability function. In the following, the central task is how to extract the Isgur–Wise function in the calculation of the CQM model.

In the CQM model, the Feynman diagram corresponding to the hadronic matrix element $\langle D_s^{(*)}|\bar{c}\gamma^\mu(1-\gamma_5)b|B_s\rangle$ is depicted in Fig. 1.

According to the CQM model [22], the couplings of $D_s(1968)$, $D_s^*(2112)$ and B_s with the light and heavy quarks are expressed by

$$\frac{1+\not{v}}{2}\sqrt{Z_H M_{D_s}}\,\gamma_5\,, \tag{10}$$

$$\frac{1+\not{v}}{2}\sqrt{Z_H M_{D_s^*}}\,\not{\epsilon}\,, \tag{11}$$

$$\frac{1+\not{v}}{2}\sqrt{Z_H M_{B_s}}\,\gamma_5\,, \tag{12}$$

where ϵ denotes the polarization vector of $D^*(2112)$. The concrete expression of Z_H is given in [22] by

$$Z_H^{-1} = (\Delta_H + m_s)\frac{\partial\mathcal{I}_3(\Delta_H)}{\partial\Delta_H} + \mathcal{I}_3(\Delta_H)\,, \tag{13}$$

$$\begin{aligned}\mathcal{I}_3(a) = &\frac{\mathrm{i}N_c}{16\pi^4}\int_{1/\Lambda^2}^{1/\mu^2}\frac{\mathrm{d}y}{y^{3/2}}\exp\left[-y\left(m_s^2 - a^2\right)\right] \\ &\times\left(1+\mathrm{erf}(a\sqrt{y})\right),\end{aligned} \tag{14}$$

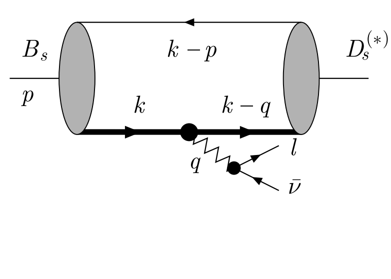

Fig. 1. Feynman diagram depicting the decays $B_s \to D_s^{(*)}l\bar{\nu}$. The *thick line* denotes the heavy quark propagator

where erf is the error function. m_s is the mass of the s quark.

Now we can write out the hadronic transition matrix element:

$$\langle D_s^{(*)}|\bar{c}\gamma^\mu(1-\gamma_5)b|B_s\rangle = \frac{(\mathrm{i}^5)}{4}\sqrt{M_{B_S}M_{D_s^{(*)}}}\,Z_H N_c \times \int^{\mathrm{reg}} \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\mathrm{Tr}\left[(\not k+m)\,\Gamma\,(1+\not v')\gamma^\mu(1+\not v)\gamma^5\right]}{(k^2-m_s^2)(v'\cdot k+\Delta_H)(v\cdot k+\Delta_H)}\,, \tag{15}$$

with $N_c = 3$, where Γ should be taken as γ_5 and $\not\epsilon$ corresponding to D_s and D_s^* respectively.

We omit technical details in the text for saving space and finally get the Isgur–Wise function $\xi(\omega)$ by comparing the results of (15) to (8) and (9):

$$\xi(\omega) = Z_{\mathrm{H}}\left[\frac{2}{1+\omega}\mathcal{I}_3(\Delta_{\mathrm{H}}) + \left(m_s + \frac{2\Delta_{\mathrm{H}}}{1+\omega}\right) \times\mathcal{I}_5(\Delta_{\mathrm{H}},\Delta_{\mathrm{H}},\omega)\right], \tag{16}$$

where the definitions of $\mathcal{I}_{1,5}$ are listed in the appendix.

By (7)–(9), we obtain explicit expressions for the semileptonic decays $B_s \to D_s(1968)l\bar{\nu}$ and $B_s \to D_s^*(2112)l\bar{\nu}$. We have

$$\begin{aligned}
&\mathrm{d}\Gamma(B_s \to D_s(1968)l\bar{\nu}) \\
&= \frac{G_{\mathrm{F}}^2|V_{cb}|^2}{48\pi^3}(M_{B_s}+M_{D_s})^2 M_{D_s}^3(\omega^2-1)^{3/2}\xi^2(\omega)\,\mathrm{d}\omega\,,\\
&\mathrm{d}\Gamma(B_s \to D_s^*(2112)l\bar{\nu}) \\
&= \frac{G_{\mathrm{F}}^2|V_{cb}|^2}{48\pi^3}(M_{B_s}-M_{D_s}^*)^2 M_{D_s}^{*3}\sqrt{\omega^2-1}(\omega+1)^2 \\
&\left[1+\frac{4\omega}{(1+\omega)}\frac{q^2}{(M_{B_s}-M_{D_s}^*)^2}\right]\xi(\omega)^2\,\mathrm{d}\omega\,,
\end{aligned} \tag{17}$$

where $q^2 = M_B^2 + M_D^2 - 2M_B M_D\omega$.

2.2 Calculation of $B_s \to D_{sJ}^*(2317)l\bar{\nu}$ and $B_s \to D_{sJ}(2460)l\bar{\nu}$ in the CQM model

The hadronic matrix elements of the decays $B_s \to D_{sJ}^*(2317)l\bar{\nu}$ and $B_s \to D_{sJ}(2460)l\bar{\nu}$ in HQET read [35]

$$\langle D_{sJ}^*(2317)|\bar{c}\gamma_\mu(1-\gamma_5)b|B_s(v)\rangle = 2\sqrt{M_{B_s}M_{D_{sJ}}(2317)}\,(v'_\mu - v_\mu)\,\zeta(\omega)\,, \tag{18}$$

$$\langle D_{sJ}(2460)\,(v',\epsilon')\,|\bar{c}\gamma_\mu(1-\gamma_5)b|B_s(v)\rangle = \sqrt{M_{B_s}M_{D_{sJ}(2460)}}\left\{2\mathrm{i}\varepsilon_{\mu\alpha\beta\gamma}\epsilon^{*\alpha}v'^{\beta}v^{\gamma} +2\left[(1-\omega)\,\epsilon_\mu^* + (\epsilon^*\cdot v)v'_\mu\right]\right\}\zeta(\omega)\,, \tag{19}$$

where $\zeta(\omega)$ denotes the Isgur–Wise function.

We use the same treatment in Sect. 2.1 to get $\zeta(\omega)$. In the CQM model, the coupling of $D_{sJ}^*(2317)(0^+)$ and $D_{sJ}(2460)(1^+)$ with the light and heavy quarks are respectively

$$\frac{1+\not v}{2}\sqrt{Z_S M_{D_{sJ}^*(2317)}}\,, \tag{20}$$

$$\frac{1+\not v}{2}\sqrt{Z_S M_{D_{sJ}(2460)}}\,\not\epsilon'\gamma_5\,, \tag{21}$$

with

$$Z_S^{-1} = (\Delta_S + m_s)\frac{\partial\mathcal{I}_3(\Delta_S)}{\partial\Delta_S} + \mathcal{I}_3(\Delta_S)\,, \tag{22}$$

where ϵ' is the polarization vector of $D_{sJ}(2460)$.

Thus, in the CQM model, the hadronic transition matrix elements of $\langle D_{sJ}^*(2317)|\bar{c}\gamma_\mu(1-\gamma_5)b|B_s(v)\rangle$ and $\langle D_{sJ}(2460)(v',\epsilon')|\bar{c}\gamma_\mu(1-\gamma_5)b|B_s(v)\rangle$ can be obtained by replacing Γ in (15) with 1 and $\not\epsilon'\gamma_5$ respectively. Meanwhile, Z_H should be replaced by $\sqrt{Z_H Z_S}$.

Finally, we get the Isgur–Wise function $\zeta(\omega)$ in the following form:

$$\zeta(\omega) = \frac{\sqrt{Z_{\mathrm{H}}Z_S}}{2(1-\omega)}\left[\mathcal{I}_3(\Delta_S) - \mathcal{I}_3(\Delta_{\mathrm{H}}) +(\Delta_{\mathrm{H}} - \Delta_S + m_s(1-\omega))\mathcal{I}_5(\Delta_{\mathrm{H}},\Delta_S)\right]. \tag{23}$$

Using (7), (18) and (19), we deduce the decay widths of $B_s \to D_{sJ}^*(2317)l\bar{\nu}$ and $B_s \to D_{sJ}(2460)l\bar{\nu}$:

$$\begin{aligned}
&\mathrm{d}\Gamma(B_s \to D_{sJ}^*(2317)l\bar{\nu}) \\
&= \frac{G_{\mathrm{F}}^2|V_{cb}|^2}{12\pi^3}M_{D_{sJ}^*(2317)}^3(M_{B_s} - M_{D_{sJ}^*(2317)})^2 \\
&\times(\omega^2-1)^{3/2}\zeta^2(\omega)d\omega\,,
\end{aligned} \tag{24}$$

$$\begin{aligned}
&\mathrm{d}\Gamma(B_s \to D_{sJ}(2460)l\bar{\nu}) \\
&= \frac{G_{\mathrm{F}}^2|V_{cb}|^2}{12\pi^3}M_{D_{sJ}(2460)}^3\sqrt{(\omega^2-1)}\zeta^2(\omega) \\
&\times\left[(M_{B_s}^2 + M_{D_{sJ}(2460)}^2)(5\omega^2-6\omega+1)\right. \\
&\left.-2M_{B_s}M_{D_{sJ}(2460)}(4\omega^3-5\omega^2+2\omega-1)\right]\,\mathrm{d}\omega\,.
\end{aligned} \tag{25}$$

3 Numerical results

The semileptonic decays $B^+ \to \bar{D}^{(*)0}l^+\nu$ and $B^0 \to D^{(*)-}l^+\nu$ are measured well [36]. The results calculated by

Table 1. The numerical results are taken from [22]

	CQM model [22]	Experiment [36]
$\mathrm{BR}[B^+ \to \bar{D}^0 l^+\nu]$	(2.2−−3.0)%	$(2.15\pm0.22)\%$
$\mathrm{BR}[B^0 \to D^- l^+\nu]$		$(2.12\pm0.20)\%$
$\mathrm{BR}[B^+ \to \bar{D}^{*0} l^+\nu]$	(5.9−−7.6)%	$(6.5\pm0.5)\%$
$\mathrm{BR}[B^0 \to D^{*-} l^+\nu]$		$(5.35\pm0.20)\%$

Table 2. The values of Δ_S and Δ_H are taken from [32]. According to (13) and (22), one gets the values of Z_S and Z_H. BR_1, BR_2, BR_3 and BR_4 respectively denote the the branching ratios of $B_s \to D_s(1968)l\bar{\nu}$, $B_s \to D_s^*(2112)l\bar{\nu}$, $B_s \to D_{sJ}^*(2317)l\bar{\nu}$ and $B_s \to D_{sJ}(2460)l\bar{\nu}$

Δ_H (GeV)	Δ_S (GeV)	Z_H (GeV)$^{-1}$	Z_S (GeV)$^{-1}$	BR_1	BR_2	BR_3	BR_4
0.5	0.86	3.99	2.02	2.95%	7.66%	5.71×10^{-3}	8.69×10^{-3}
0.6	0.91	2.69	1.47	2.86%	7.53%	5.25×10^{-3}	7.91×10^{-3}
0.7	0.97	1.74	0.98	2.73%	7.49%	4.90×10^{-3}	7.52×10^{-3}

Table 3. In this table, we list our results for the semileptonic decays of B_s to $D_s(1968)$, $D_s^*(2112)$, $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ and that obtained by other approaches [19–21]

Processes	CQM	QSR in HQET ($m_{\mathrm{Q}} \to \infty$) [19]	QSR
$\mathrm{BR}[B_s \to D_s(1968)l\bar{\nu}]$	(2.73–3.00)%	–	–
$\mathrm{BR}[B_s \to D_s^*(2112)l\bar{\nu}]$	(7.49–7.66)%	–	–
$\mathrm{BR}[B_s \to D_{sJ}^*(2317)l\bar{\nu}]$	$(4.90–5.71) \times 10^{-3}$	0.09	$\sim 10^{-3}$ [20]
$\mathrm{BR}[B_s \to D_{sJ}(2460)l\bar{\nu}]$	$(7.52–8.69) \times 10^{-3}$	0.08	4.9×10^{-3}[21]

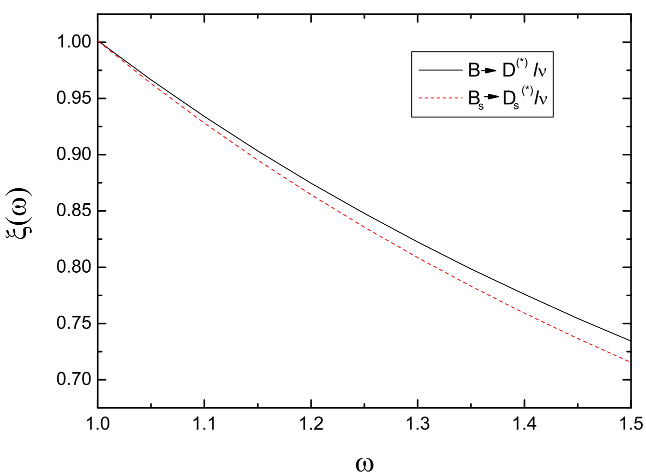

Fig. 2. The dependence of the Isgur–Wise function $\xi(\omega)$ on ω with the typical values of $\Delta_H = 0.3$ GeV and $\Delta_H = 0.5$ GeV corresponding to the $B \to D^{(*)}l\nu$ and $B_s \to D_s^{(*)0}l\nu$ decays respectively

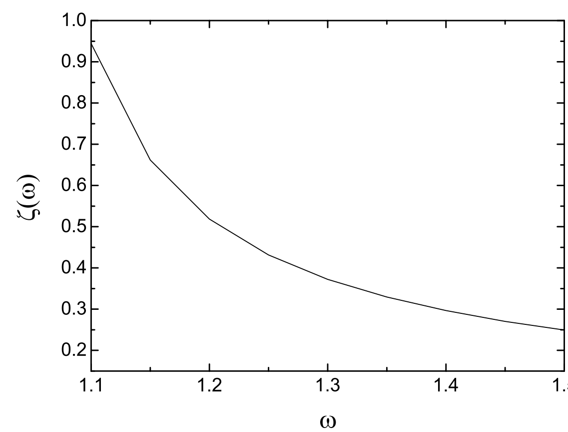

Fig. 3. The dependence of $\zeta(\omega)$ on ω with the typical values of $\Delta_H = 0.5$ GeV and $\Delta_S = 0.86$ GeV corresponding to the $B_s \to D_{sJ}(2317, 2460)l\bar{\nu}$ decays

the CQM model [22] and measured by the experiments are collected in Table 1. By comparing the theoretical results with those measured by the experiments, one is led to believe that the CQM model is applicable to our processes, and one expects to get a relatively reliable result.

With the formulation derived from the last section, one numerically evaluates the corresponding decay rates. The input parameters include $G_{\mathrm{F}} = 1.1664 \times 10^{-5}\,\mathrm{GeV}^{-2}$, $V_{cb} = 0.043$, $M_{B_s} = 5.3696$ GeV, $M_{D_s} = 1.968$ GeV, $M_{D_s^*} = 2.112$ GeV, $M_{D_{sJ}^*(2317)} = 2.317$ GeV, $M_{D_{sJ}(2460)} = 2.46$ GeV [36]. $m_s = 0.5$ GeV, $\Lambda = 1.25$ GeV, the infrared cutoff $\mu = 0.593$ GeV and $\Delta_S - \Delta_H = 335 \pm 35$ MeV [32].

We present the branching ratios of $B_s \to D_s(1968)l\bar{\nu}$, $B_s \to D_s^*(2112)l\bar{\nu}$, $B_s \to D_{sJ}^*(2317)l\bar{\nu}$ and $B_s \to D_{sJ}(2460)l\bar{\nu}$ in Table 2.

In Fig. 2, we show the dependence of the Isgur–Wise function $\xi(\omega)$ on ω in the decays $B \to D^{(*)}l\nu$ and $B_s \to D_s^{(*)0}l\nu$. The dependence of $\zeta(\omega)$ on ω is shown in Fig. 3.

For comparison, we also display the values of the branching ratios of $B_s \to D_s(1968)l\bar{\nu}$, $B_s \to D_s^*(2112)l\bar{\nu}$, $B_s \to D_{sJ}^*(2317)l\bar{\nu}$ and $B_s \to D_{sJ}(2460)l\bar{\nu}$, which are calculated in [20, 21], in Table 3.

4 Discussion and conclusion

By comparison between the theoretical results and experimental results listed in Table 1, we have reason to believe that the CQM model can well be applied to the study of the semileptonic decays related to this work. In this work, we study the semileptonic decays of B_s to $D_s(1968)$, $D_s^*(2112)$, $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$. In HQET, $D_s(1968)$ and $D_s^*(2112)$ can be well categorized as the H doublet $(0^-, 1^-)$. To calculate the semileptonic decays relevant to $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$, we make the assumption that $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ belong to the S doublet $(0^+, 1^+)$.

At present the experiments only give $\mathrm{BR}[B^0_s \to D^-_s l^+ \nu_l$ anything$] = (7.9 \pm 2.4)\%$ [36], and information on $B_s \to D_s(1968)l\overline{\nu}$ is still absent. Thus the result of $B_s \to D_s(1968)l\overline{\nu}$ by the CQM model does not contradict this experimental value. Meanwhile one predicts the branching ratio of $B_s \to D^*_s(2112)l\overline{\nu}$, whose order of magnitude is the same as that of $B_s \to D_s(1968)l\overline{\nu}$. The semileptonic decay $B_s \to D^*_s(2112)l\overline{\nu}$ should be seen in future experiments. In Fig. 2, the dependence of the Isgur–Wise function of $B_s \to D^{(*)0}_s l\nu$ on ω is very similar to that of $B \to D^{(*)}l\nu$. The difference between them comes from the breaking of SU(3) symmetry and HQET symmetry. Thus, we believe that the model is applicable to our processes and expect to obtain relatively reliable results.

The branching ratios of $B_s \to D_{sJ}(2317, 2460)l\overline{\nu}$ from our calculations and that obtained by QCD sum rules [20, 21] are of the same order of magnitude. However, our predictions are far smaller than those given by [19]. Anyway, at present both our calculations and the analyses from other groups all indicate that the semileptonic decays $B_s \to D^*_{sJ}(2317, 2460)l\overline{\nu}$ have large branching ratios. Therefore we urge our experimental colleagues to measure those semileptonic channels in the CDF experiment and in the future LHCb experiment. It will help us to further understand the nature of those exotic D_{sJ} mesons. And doing more experiments in future will also improve our understanding of the models applied to calculation of the semileptonic decays of the B_s to D_{sJ} mesons.

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Appendix

The explicit expressions of $\mathcal{I}_{1,5}$ that are related to our calculation are listed here:

$$\mathcal{I}_1 = \frac{\mathrm{i}N_c}{16\pi^4}\int^{\mathrm{reg}} \frac{\mathrm{d}^4k}{k^2 - m_s^2} = \frac{N_c m_s^2}{16\pi^2}\Gamma\left(-1, \frac{m_s^2}{\Lambda^2}, \frac{m_s^2}{\mu^2}\right), \tag{A.1}$$

$$\begin{aligned}
&\mathcal{I}_5(\alpha_1, \alpha_2, \omega) \\
&= \frac{\mathrm{i}N_c}{16\pi^4}\int^{\mathrm{reg}} \frac{\mathrm{d}^4k}{(k^2 - m^2)(v\cdot k + \alpha_1 + \mathrm{i}\epsilon)(v'\cdot k + \alpha_2 + \mathrm{i}\epsilon)} \\
&= \int_0^1 \mathrm{d}x \frac{1}{1 + 2x^2(1-\omega) + 2x(\omega - 1)} \\
&\quad \times \left[\frac{6}{16\pi^{3/2}}\int_{1/\Lambda^2}^{1/\mu^2} \mathrm{d}s\, \varrho\, \mathrm{e}^{-s(m_s^2 - \varrho^2)} s^{-1/2}\left(1 + \mathrm{erf}\left(\varrho\sqrt{s}\right)\right)\right. \\
&\quad \left. + \frac{6}{16\pi^2}\int_{1/\Lambda^2}^{1/\mu^2} \mathrm{d}s\, \mathrm{e}^{-s(m_s^2 - 2\varrho^2)} s^{-1}\right],
\end{aligned} \tag{A.2}$$

where the definitions of $\mathrm{erf}(z)$ and ϱ are

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z \mathrm{d}x\, \mathrm{e}^{-x^2}, \tag{A.3}$$

$$\varrho = \frac{\alpha_1(1-x) + \alpha_2 x}{\sqrt{1 + 2(\omega - 1)x + 2(1-\omega)x^2}}. \tag{A.4}$$

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